

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Sum the series to infinity.

$$\frac{15}{16} + \frac{15.21}{16.24} + \frac{15.21.27}{16.24.32} + \dots$$

17. Solve $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.

18. Solve the equation

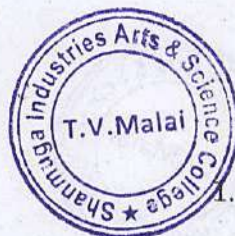
$$x + 2y + 3z = 0$$

$$2x + y + 3z = 0$$

$$3x + 2y + z = 0$$

19. If $\tan \frac{x}{2} = \tanh \frac{x}{2}$, prove that $\cos x \cosh x = 1$.

20. If $i^{\alpha+i\beta} = \alpha + i\beta$, prove that $\alpha^2 + \beta^2 = e^{-\pi\beta}$.



NOVEMBER/DECEMBER 2019

BAEL12 — BASIC MATHEMATICS — I

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

- Write down the middle term of $\left(x - \frac{2}{x}\right)^{12}$.
- If $|x| > 1$, prove $\frac{1}{x} + \frac{1}{3x^2} + \frac{1}{5x^4} + \dots = \log \sqrt{\frac{x+1}{x-1}}$.
- If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ find the condition if $\alpha\beta = -1$.
- Diminish by 3 the roots at $x^4 + 3x^3 - 2x^2 - 4x - 3 = 0$.
- If $A = \begin{bmatrix} 2 & 5 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 8 \\ -1 & 3 \end{bmatrix}$ find $2A - 3B$.
- If $A = \begin{bmatrix} 3 & -2 & 0 \\ 1 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 2 & 3 \\ 4 & 1 & -1 \end{bmatrix}$ find AB .
Does BA exist?

7. Prove that

$$(\sin x + i \cos x)^n = \cos n \left(\frac{\pi}{2} - x \right) + i \sin n \left(\frac{\pi}{2} - x \right).$$

8. Evaluate $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$.

9. Prove that $\cosh 2x = 1 + 2 \sinh^2 x$.

10. Find the expression for $\log(-2)$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Resolve into partial fractions $\frac{9}{(x-1)(x+2)^2}$.

Or

- (b) Sum the series $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \dots \infty$.

12. (a) Solve the equation

$$x^4 - 2x^3 + 4x^2 + 6x - 21 = 0 \text{ given that two of its roots are equal in magnitude and opposite in sign.}$$

Or

- (b) Remove the fractional co-efficient from the equation $x^3 + \frac{1}{4}x^2 - \frac{1}{6}x + \frac{1}{72} = 0$.

13. (a) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}.$$

Or

- (b) Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}.$$

14. (a) Prove that

$$(1+i\sqrt{3})^n + (1-i\sqrt{3})^n = 2^{n+1} \cos \frac{n\pi}{3}.$$

Or

- (b) Prove that

$$\cos^5 \theta = \frac{1}{16} [\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta].$$

15. (a) Prove that $\tanh^{-1} x = \frac{1}{2} \log \left(\frac{1+x}{1-x} \right)$.

Or

- (b) If $\cos^{-1}(\alpha + i\beta) = \theta + i\phi$ show that

$$\alpha^2 \operatorname{sech}^2 \phi + \beta^2 \operatorname{cosech}^2 \phi = 1.$$

